JOHN CURTIN
College of the Arts
COLLEGE OF THE ARTS

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MATHEMATICS METHODS YEAR 11

TEST 3, 2018

(Trigonometric Functions, Probability)

SECTION ONE: CALCULATOR-FREE

Student's Name: Mark Kinké

Total Marks: 24
Time Allowed: 25 minutes

MATERIAL REQUIRED/RECOMMENDED FOR THIS TEST

Standard Items: Pens, pencils, eraser, ruler

Special Items: Formula Sheet

IMPORTANT NOTE TO STUDENTS:

The standard items listed above, a Formula Sheet provided by the supervisor and this Calculator-Free test-paper are the only items permitted on your desk during the Calculator-Free test. When directed, you will place all other items under your chair.

INSTRUCTIONS TO STUDENTS

Do not open this paper until instructed to do so.

You are required to answer ALL questions.

Write answers in the space provided beneath each question.

Marks are shown with the questions.

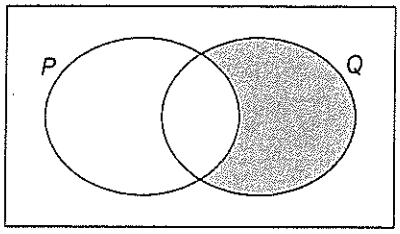
Show all working clearly, in sufficient detail to allow your answers to be checked and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks.

It is recommended that students **do not use pencil**, except in diagrams.

Question 1 [2, 1, 1, 1 = 5 marks]

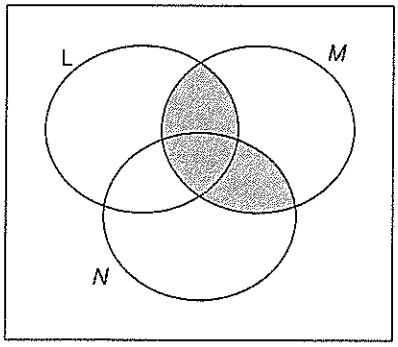
(a) Describe the shaded region in set notation.

(i)



$P' \cap Q$ ✓

(ii)



$M \cap (L \cup N)$ *or* $(L \cap M) \cup (M \cap N)$ ✓

b) U = {positive integers between 1 and 20, where 1 and 20 are not included}

A = {prime numbers less than 19} B = {factors of 12} C = {multiples of 3}

$A = \{2, 3, 5, 7, 11, 13, 17\}$ $B = \{2, 3, 4, 6, 12\}$ $C = \{3, 6, 9, 12, 15, 18\}$

State the following:

(i) $A \cap B \cap C$

$A \cap B \cap C = \{3\}$ ✓

(ii) $(A \cup B)'$

$A \cup B = \{2, 3, 4, 5, 6, 7, 11, 12, 13, 17\}$

$(A \cup B)' = \{8, 9, 10, 14, 15, 16, 18, 19\}$ ✓

(iii) $n(B \cup C)$

$n(B \cup C) = 8$ ✓

(2, 3, 4, 6, 12, 9, 15, 18)

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Question 2 [2, 4 = 6 marks]

(a) Find the exact value of $(\sin 250°)(\cos 25°) - (\cos 250°)(\sin 25°)$.

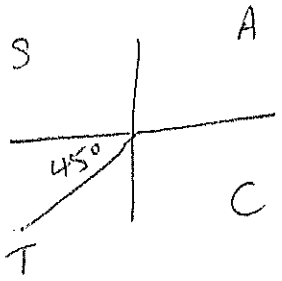
$\sin(250°)\cos(25) - \cos(250)\sin(25)$

$= \sin(250 - 25)$

$= \sin 225°$ ✓

$= -\sin 45°$ $= -\frac{1}{\sqrt{2}}$

$= -\frac{\sqrt{2}}{2}$ ✓



(b) Use the sum of two angles to find the exact value of $\tan\frac{7\pi}{12}$.
Rationalise the denominator of your solution.

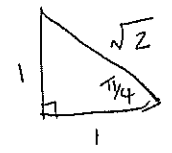
$\frac{\pi}{4} \rightarrow 45°$

$\frac{\pi}{3} \rightarrow 60°$

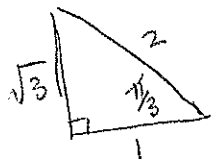
$\tan\left(\frac{3\pi}{12} + \frac{4\pi}{12}\right) = \tan\left(\frac{\pi}{4} + \frac{\pi}{3}\right)$ ✓

$= \frac{\tan\frac{\pi}{4} + \tan\frac{\pi}{3}}{1 - \left(\tan\frac{\pi}{4}\right)\left(\tan\frac{\pi}{3}\right)}$ ✓

$= \frac{1 + \sqrt{3}}{1 - \sqrt{3}}$ ✓



$\tan\frac{\pi}{4} = 1$



$\tan\frac{\pi}{3} = \frac{\sqrt{3}}{1}$

$= \sqrt{3}$

$= \frac{(1+\sqrt{3})}{(1-\sqrt{3})} \cdot \frac{(1+\sqrt{3})}{(1+\sqrt{3})}$

$= \frac{4 + 2\sqrt{3}}{-2}$

$= -2 - \sqrt{3}$ ✓

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Question 3 **[1, 1, 1, 2 = 5 marks]**

A doctor wishes to tell her patient's family the probability of his condition improving after a certain treatment. Suppose I is the event that the patient's condition improves, O is the event that his condition remains the same and W is the event that his condition worsens.
If P(W) = 30% and P(O) = 10 % find the probability that the patient's condition:

P(I) = 60%.

(a) improves.

60%

$$P(I) = 100\% - P(W) - P(O)$$
$$= 100\% - 30\% - 10\%$$
$$= 60\% \checkmark$$

(b) does not improve.

40%

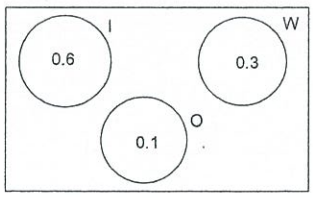
$$P(\bar{I}) = 100\% - P(I)$$
$$= 100\% - 60\%$$
$$= 40\% \checkmark$$

(c) improves or stays the same.

70%

$$P(I \cup O) = P(I) + P(O)$$
$$= 60\% + 10\%$$
$$= 70\% \checkmark$$

(d) Draw a Venn diagram which shows that these three events are mutually exclusive.



✓ No intersection
✓ Labels, etc.

Accept overlapping if clear prob. = 0.

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Question 4 **[4, 2, 2 = 8 marks]**

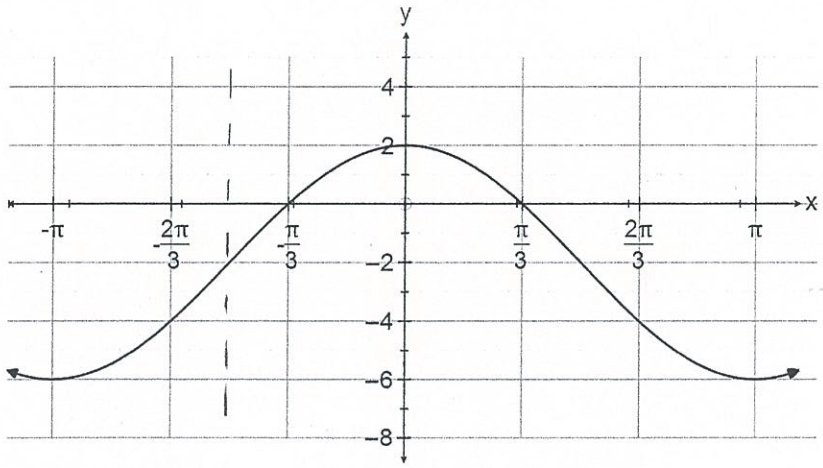
The graph below can be written $k(x) = a\cos(x) - b = c\sin(x - d) - b$

(a) Determine the values of a, b, c and d.

$\frac{\pi}{2} \Rightarrow \frac{3\pi}{6}$

$\frac{2\pi}{3} \Rightarrow \frac{4\pi}{6}$

$\frac{\pi}{3} = \frac{2\pi}{6}$



$4\cos(x) - 2 = -4\sin\left(x - \frac{\pi}{2}\right) - 2$

$a = 4 \quad b = 2 \quad c = -4 \quad d = \frac{\pi}{2}$ or $c = 4\ d = -\frac{\pi}{2}$ ✓✓✓✓

Amplitude $= \frac{2 - -6}{2} = 4 \Rightarrow a = 4$

Translation 2 unit down $\Rightarrow b = 2$

$\cos(x) = \sin(x - \pi/2) \Rightarrow d = \pi/2$

sine will be reflection in x-axis $\Rightarrow c = -4$

Accept $c = 4$ without (d)

(b) State the range of $k(x)$.

$R_{k(x)} = \{y : -6 \leq y \leq 2, y \in \mathbb{R}\}$ ✓✓

(c) Explain why the graph represents a function and verify your explanation by using the vertical line test on the graph.

For every x value there is only one y value ✓

Vertical line through graph ✓

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MATHEMATICS METHODS YEAR 11

TEST 3, 2018

(Trigonometric Functions, Probability)

SECTION TWO: CALCULATOR-ASSUMED

Student's Name: Mark Kinké

Total Marks: 29

Time Allowed: 30 minutes

MATERIAL REQUIRED/RECOMMENDED FOR THIS TEST

Standard Items: Pens, pencils, eraser, ruler

Special Items: Up to three approved calculators

One page (unfolded A4 sheet) front and back of Notes

Formula Sheet retained from Section One

INSTRUCTIONS TO STUDENTS

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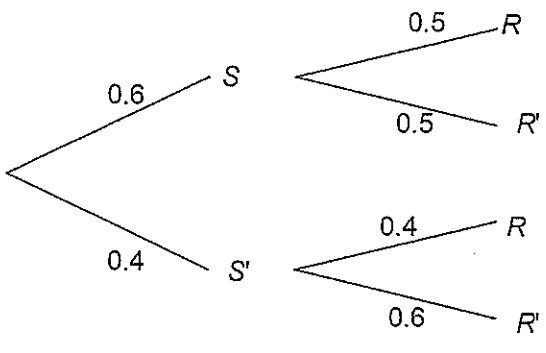
Question 5 **[3, 1, 1, 2, 2 = 9 marks]**

Given the following: $P(R|S) = 0.5$, $P(R|S') = 0.4$ and $P(S) = 0.6$

(a) Complete the tree diagram showing all the probabilities.



SR

SR'

S'R

S'R'

✓✓✓

−1 per error.

(b) Determine the following:

(i) $P(S \cap R)$ = 0.6×0.5

0.3 ✓

(ii) $P(S' \cap R')$ = 0.4×0.6

0.24 ✓

(iii) $P(R)$ = $0.6 \times 0.5 + 0.4 \times 0.4$ ✓

= $0.3 + 0.16$

0.46 ✓

(iv) $P(S'|R)$ = $\frac{0.4 \times 0.4}{0.46} = \frac{0.16}{0.46}$ ✓

0.348 or $\frac{16}{46}$ ✓

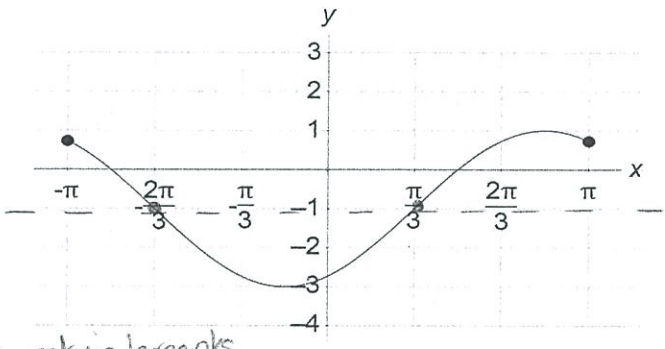
Question 6 **[2, 3, 1 = 6 marks]**

(a) Write the equation of a sine function with the following features:
an amplitude of 2 $\Rightarrow a = 2$
a phase shift of $\frac{\pi}{3}$ to the right $\Rightarrow c = \pi/3$
a period of 2π $\Rightarrow b = 1$
a vertical translation of 1 in the negative direction $\Rightarrow d = -1$

$y = a\sin(bx - c) + d$

$y = 2\sin\left(x - \frac{\pi}{3}\right) - 1$ ✓✓

(b) Draw the function from (a) on the axes below for the domain $x \in [-\pi, \pi]$:



✓✓✓ Correct intercepts
Correct max/min
Smooth curve.

(c) $2\sin\left(x - \pi/3\right) - 1 = -1$
$\Rightarrow 2\sin\left(x - \pi/3\right) = 0$

(c) Use the graph from (b), or otherwise, to find the solutions to $2\sin\left(x - \frac{\pi}{3}\right) = 0$ in the given domain.

$x = -\frac{2\pi}{3}$ or $\frac{\pi}{3}$ ✓

-2.094 1.047

Solutions from intersections between $y = 2\sin(x - \pi/3) - 1$ and $y = -1$

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Question 7 **[3, 1, 1, 1, 1, 2, 1 = 10 marks]**

G and H are events such that $P(G) = 0.7$, $P(G \cap H) = 0.4$ and $P(G' \cap H) = 0.2$.

(a) Complete the two-way table below.

	H	H'	
G	0.4	0.3	0.7
G'	0.2	0.1	0.3
	0.6	0.4	1

✓✓✓ −1 per error.

(b) Find:

(i) $P(G \cap H')$

0.3 ✓

(ii) $P(H)$

0.6 ✓

(iii) $P(G' \cap H')$

0.1 ✓

(iv) $P(G \cup H)$ $= P(G) + P(H) - P(G \cap H)$

0.9 ✓ $= 0.7 + 0.6 - 0.4$ ✓

(v) $P(G|H)$ $= \dfrac{P(G \cap H)}{P(H)} = \dfrac{0.4}{0.6} = \dfrac{4}{6}$ ✓

$\frac{2}{3}$ ✓

(c) Show that G and H are not mutually exclusive.

Not mutually exclusive: $P(G \cap H) = 0.4$ Should be 0 ✓

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Question 8 [4 marks]

Given that $\cos\theta = -\frac{8}{15}$ and $\tan\theta$ is positive, find the exact value of $\cos 2\theta$.

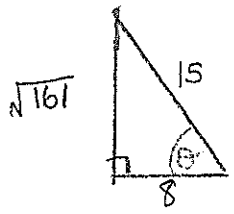
$\sin\theta = -\frac{\sqrt{161}}{15}$ ✓

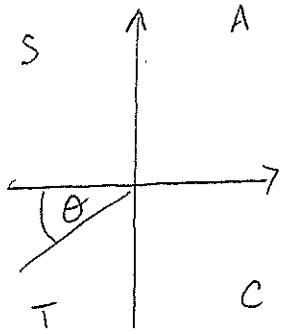
$(\cos(\theta+\theta))$

$\cos 2\theta = (\cos\theta)(\cos\theta) - (\sin\theta)(\sin\theta)$ ✓

$= \frac{64}{225} - \frac{161}{225}$ ✓ $\quad -\frac{8}{15}\times\frac{-8}{15} - \frac{\sqrt{161}}{15}\times\frac{\sqrt{161}}{15}$

$= -\frac{97}{225}$ ✓





tan θ + ve ⇒ sinθ -ve
cosθ - ve

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